# Additional material <br> Tracing Field-Coherent Quad Layouts <br> paper 1125 <br> Extraction of separatrix directions on singularities 

## 1 Extraction of separatrix directions on singularities

As shown in Figure 1, we consider a vertex $v$ and a cross field defined on each surrounding face and we want to derive the directions of the separatrices on $v$ (Fig. 1.a).
We assume the cross field do be defined exactly in the barycenter $B_{i}$ of each face. We define the set of directions that span between two adjacent faces by linearly interpolating the directions of their crosses placed at the two barycenters. Intuitively, a separatrix appears when one of these spanned interpolated directions hits the vertex (Fig. 1.b).
In practice, for each pair of consecutive faces $f_{0}, f_{1}$ we extract a pair of coherent directions $u_{0}$ and $u_{1}$, then we test if $u_{0}$ and $u_{1}$ rotate in opposite directions in order to parallel match the vectors $\left(B_{0}-v\right)$ and $\left(B_{1}-v\right)$, respectively. In this case we derive the direction $s$ by interpolating between $u_{0}$ and $u_{1}$ considering the angles $a_{0}$ and $a_{1}$ shown in Figure 1.c. If the test with $u_{0}$ and $u_{1}$ fails, we perform the same test with the other pair of orthogonal directions of the cross field.
In order to avoid that the two directions $\left(B_{0}-v\right)$ and $\left(B_{1}-v\right)$ span an angle $\gamma>90^{\circ}$ (introducing inconsistencies in the evaluation of separatrices in a 4-rosy field), then we split $\gamma$ into three equivalent angles, defining three triangles (see Figure 1.d). We interpolate the field for each sub triangle, then we perform the separatrix test for each pair of consecutive sub triangles.


Figure 1: Derivation of separatrix directions on a singularity: (a) The initial cross field and the cross field set at triangle's barycenter; (b) A separatrix direction is found when the set of directions obtained spanning between $u_{0}$ and $u_{1}$ meet the singularity; (c) This test is performed by checking if $\left(B_{0}-v\right) \times u_{0}$ and $\left(B_{1}-v\right) \times u_{1}$ have opposite directions, the direction $s$ is obtained by interpolating between $a_{0}$ and $a_{1}$; (d) Each face is splitted in three sub-faces to avoid the angle between $\left(B_{0}-v\right)$ and $\left(B_{1}-v\right)$ to be greater then $90^{\circ}$.

